A Power-Law of Death

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Introduction

• The distribution of executions across the countries of the world, US states, and US counties corresponds to what statisticians call a “power law.”

• This means that the relation between the cumulative frequency of the event and the severity of the event follows this equation: \( F(x) = ab^{-\alpha} \)

• Where \( F(x) \) means the cumulative frequency of \( x \); \( a \) is a constant, \( b \) is the severity, and \( \alpha \) is a parameter to be estimated.
What that means

• If one plots the frequency of the event against the severity of the event, and uses a log scale for both the x and the y axis, a series that has a “power law” characteristic will array along a straight line.

• Power-laws are “extreme value” distributions in which theories suggest a “self-reinforcing” process must be generating the distribution.

• If events occurred randomly, they could not generate such a distribution.
Implications

• What process could produce a “self-reinforcing” outcome for executions?
• Local legal communities may never or very rarely generate executions, in which case norms and procedures develop to “self-reinforce” this abstention from capital punishment.
• Or, they may start down the path, and when they do the path becomes easier in subsequent cases.
• If the distribution of executions is a power-law, it suggests something of this nature.
• A self-reinforcing legal culture is strong evidence of arbitrariness, since the odds of execution are related to the number of previous executions in that jurisdiction, not the characteristics of the crime.
The Data

• Data compiled on all US executions since 1976 as of April 11 2011, 1245 executions in total.
• Analyses by county do not include 3 executions by the federal government.
• Subsequent slides show for states and counties the distribution, first as simple counts, then on a log-log plot to test for the presence of a power law. This is also shown then for successive time periods from 1977 forward, for counties. Finally we consider all countries in the world, from 2007 to 2010. The power law is ubiquitous.
• Thanks to UNC undergraduate students BJ Dworak, Matt Nolan, Linden Wait, and Amber Clifford for research assistance.
Number of Executions across US States

Includes 50 states, DC, and Federal Government
A Power Law of Death across the 50 States

Includes 50 states, DC, and Federal Government

Ln (Executions + 1) = 6.42 - 1.414(Ln (Frequency)) Adj. $R^2 = 0.911$
Executions by County

Includes 1245 executions from 1977 to April 10 2011. 2692 counties have executed no inmates at least one and Harris County 116.
US counties with 10 or more executions since 1977

Includes counties with 10 or more executions from 1977 to April 10, 2011.
Number of Executions across US Counties

454 counties executed at least one person but one executed more than 100
A Power Law of Death

\[ \ln(\text{Executions} + 1) = 4.12 - 0.569 \ln(\text{Frequency}) \]

Adj. \( R^2 = 0.95 \)
Executions by county 1977 to 1985

Number of Counties

Number of Executions

40 counties 50 executions
Executions by county 1977 to 1990

Number of Counties

Number of Executions

89 counties 143 executions
Executions by county 1977 to 1995

Number of Counties

Number of Executions

169 counties 313 executions
Executions by county 1977 to 2010

Number of Counties vs. Number of Executions

454 counties 1231 executions
Executions by county 1977 to 2011

454 counties 1242 executions as of April 11
These trends also hold for individual states

- The following slides show similar analyses for the state with by far the greatest number of executions, Texas, and for North Carolina.

- We can have greater confidence in the national analysis since it is based on a larger number of observations, but the pattern also holds within individual states.
Texas counties with 5 or more executions 1977 to 2011

Note: 164 of the 254 counties in Texas have had no executions.
Texas executions by county

Of 254 counties, 164 have executed no one but Harris has executed 116.
Among 254 counties in Texas, 90 have had one or more executions, 9 counties have executed 10 or more, and one (Harris) has executed 116. 

\[
\ln(\text{Frequency}) = 4.36 - 0.85(\ln(\text{Executions}+1)) \quad \text{Adj. R}^2 = 0.97
\]
Executions by County in North Carolina 1977 to 2011

Note: 74 of the 100 counties in North Carolina have had no executions.
78 counties have executed no one but Mecklenberg has executed 5.
Among North Carolina's 100 counties, 26 have had one or more executions, 8 counties have executed 2 or more, and one (Mecklenberg) has executed 5. 

\[ \ln(\text{Frequency}) = 1.8 - 0.34(\ln(\text{Executions} + 1)) \]

Adj. R\(^2\) = 0.95
These trends also hold for countries across the world

• Since 2007, Amnesty International has published an annual review of capital punishment around the world: http://www.amnesty.org/en/death-penalty/numbers

• Where they present a range, I use the lowest number in order to be conservative.

• Following charts combine 2007 through 2010.
Executions by Country, 2007-2010

Number of Executions

Includes only countries with six or more executions.
Executions by Country, 2007-2010

- Viet Nam: 54
- North Korea: 77
- Yemen: 111
- Pakistan: 171
- USA: 177
- Iraq: 188
- Saudi Arabia: 341
- Iran: 1303
- China: 4198

Includes only countries with 50 or more executions.
Executions by country, 2007-2010

Of 196 countries, 164 executed no one but China executed over 4,000.
A Power Law of Death Across the World, 2007 to 2010

Among 196 countries in the world, 164 have had no executions, 7 have executed 100 or more, and one (China) has executed over 4,000.

\[
\ln(\text{Frequency}) = 8.62 - 2.17(\ln(\text{Executions}+1)) \quad \text{Adj. R}^2 = 0.98
\]
Other Possible Processes

• Imagine a process with multiple stages, and a fixed percentage of the cases make it through each filter. If the filter selected out 90% of the cases each time, and we started with 100, the cases would be ordered: 100, 90, 81, 72, 63, etc. with each case having 90% of the value of the previous case.

• Or imagine a process where each stage amplifies the value: say by 20%: 1, 1.2, 1.44...

• Other processes might be that all cases are equal, or random. The following graphs show what distributions such processes would generate.
If all cases were equal

Frequency Distribution

Log-Log Presentation

All values have an equal number of observations

Log-log presentation of a perfectly equal distribution
If all cases were random

**Frequency Distribution**

Pure randomness

**Log-Log Presentation**

Log-log presentation of a random distribution

Cases have average 50 with a st. dev. of 15.
Each case 90% of previous case

Frequency Distribution

Log-Log Presentation

One case has 100, 2 at least 90, 3 at least 81, 4 at least 72, but most have close to zero.

One case has 100, 2 at least 90, 3 at least 81, 4 at least 72, but most have close to zero. A power-law would array on a straight line here.
Each case 50% of previous case

Frequency Distribution

Log-Log Presentation

Each value is 50% of the previous value

Log-log presentation of 50 percent decline

One case has 100, 2 at least 50, 3 at least 25, 4 at least 12, but most have close to zero. A power-law would array on a straight line here.
Each case 25 percent of previous case

Frequency Distribution

Log-Log Presentation

Each value is 25% of the previous value

Log-log presentation of 75 percent decline

One case has 100, 2 at least 25, 3 at least 8, 4 at least 2, but most have close to zero.

One case has 100, 2 at least 25, 3 at least 8, 4 at least 2, but most have close to zero. A power-law would array on a straight line here.
20 percent growth from case to case

Frequency Distribution

Each value 20% greater than the previous one

Log-Log Presentation

Log-log presentation of 20 percent growth
20 percent growth from case to case

This distribution arrays on a straight line when we take the log of the value but not the log of the frequency. This is because the logarithm perfectly captures the concept of steady percentage growth. The slope of the line relates to the percent of growth.

Semi-Log Presentation

Semi-log presentation of 20 percent growth
So it can’t be random

- Random processes do not produce power-law distributions.

- Rather, there must be some dynamic that causes an extreme distribution: an “amplification” parameter that pushes a few cases into the extremes while preventing the vast bulk of cases from having values much above zero.

- Most likely, local legal cultures and the development of localized norms are the key.
Local Legal Cultures

• These can reinforce a culture of “no death penalty” or they can render it relatively common.

• This simple analysis has shown that such things are plausible explanations at the county, state, and global levels.

• Of course two elements remain:
  – Demonstrate statistically that the distributions are indeed extreme value, either exponential or power-law.
  – Investigate the legal cultures and histories in jurisdictions with many and few executions to see if my hypothesis is correct “on the ground.”
Comments welcome

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